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TITLE: TEXTURE AND SHEET FORMING

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# TEXTURE AND SHEET FORMING

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## ABSTRACT

A large number of studies have been made concerning the sheet forming problem, however most of them use phenomenological theories, and only few authors have used the constitutive behavior resulting from texture hardening and microscopic hardening to characterize the sheet formability. These latter studies, however, usually assume that the texture does not develop during the forming operation, and that the grains are equiaxed.

The classical Marciniak-Kuczynski (Defect) theory, which consists in calculating the behavior of an initial defect in the sheet, in the form of a thin groove, is here applied together with a full-constraints or relaxed-constraints theory of polycrystal viscoplasticity. The purpose of this is to investigate the effect of the induced texture on the Forming Limit Diagram (FLD), and the effect of grain shape as well.

An alternative fast way of deriving F.L.D.'s is also proposed using a perturbation method. Comparisons are made between the results obtained by both Defect and Perturbation theories, in the case of ideal f.c.c. rolling texture components, and in the case of polycrystals.

## INTRODUCTION

The sheet forming process carries a great industrial interest in as much as a better simulation of that operation can help avoiding material waste due mainly to flow inhomogeneities that affect the surface aspect and may also induce ears.

The tool used by engineers to characterize the expected ductility in the forming process is the Forming Limit Diagram. One way of deriving experimentally this diagram is to perform a series of tests on the same initial material on which a given loading path is imposed, and recording the critical strain at which failure occurs or at which a given strain gradient appears. Despite the fact that earing induces non-zero shear strains, all F.L.D. are represented only in a two-dimensional strain plot. Most of the authors dealing with that subject, and trying to reproduce those figures have used phenomenological relations to describe the material behavior. Only in the last decade, some authors considered using the information given by the texture and the microscopic behavior to represent the anisotropy responsible in part of the flow instabilities; among them, P.Bate [1], F.Barlat[2], R.J.Asaro and A.Needleman[3] and Kocks et al.[4]. The theoretical way used by these authors to treat this problem is the following. The material will promote a flow localization in the form of a groove having a small height and a small thickness compared to its length (Fig.1). At any time, this groove must fulfill three conditions:

$$(h\sigma_{nn})^G=(h\sigma_{nn})^B$$
$$(h\sigma_{nt})^G=(h\sigma_{nt})^B \qquad (1)$$
$$D_{tt}^G=D_{tt}^B$$

where (n, t, 3) are the groove axes, $\underset{\sim}{\sigma}$ is the local Cauchy stress tensor, $\underset{\sim}{D}$ the local strain rate tensor, and h the current local sheet thickness. The purpose of the Defect Theory used by these authors is to create an initial groove at a given initial angle by artificially setting a thickness difference of given size ( that can, for instance, represent the surface roughness of the sheet), and see how this defect evolves by applying to it at any strain the equations (1). Further realistic simplifications are also made:

$$\sigma_{33}=0 \qquad (2)$$

i.e. plane stress, and no triaxial effects due a sharp localization, i.e. a long wavelength approximation. It is also assumed that the strain path is known in the bulk, i.e.:

$$\rho=(D_{22}/D_{11})^B=\text{given} \qquad (3)$$

An arbitrary decision is usually made concerning the definition of the critical strain at which unbounded localization is considered to take place. The critical groove angle is the one leading to the minimum localization strain.

The purpose of the present work is to use the flow behavior of a viscoplastic polycrystalline material predicted by a texture model, to construct F.L.D.'s. Two approaches will be used: Defect and Perturbation theories. Texture and grain shape effects will be investigated.

The next section will be devoted to the relation between the single crystal and the polycrystal behaviors.

## SINGLE CRYSTAL vs POLYCRYSTAL BEHAVIOR

A single crystal is assumed to deform plastically on well defined slip systems s characterized by their slip plane $\underset{\sim}{n}^s$, and their slip direction $\underset{\sim}{b}^s$. When a deviatoric stress $\underset{\sim}{S}$ is applied to the single crystal, the slip system will glide at a microscopic shearing rate $\dot{\gamma}^s$ so that:

$$\dot{\gamma}^s=\dot{\gamma}_0(m^s{}_{ij}S_{ij}/\tau^s{}_0)^{m'} \qquad (4)$$

where $\dot{\gamma}_0$ and $\tau_0{}^s$ are respectively the reference shear strain rate and the reference stress associated to .t. The parameter m' is the inverse of the rate-sensitivity factor. The tensor $m^s$ is defined as:

$$m^s{}_{ij}=b^s{}_i n^s{}_j \qquad (5)$$

and the product $\underset{\sim}{m}^s:\underset{\sim}{S}$ is the resolved shear stress on slip system s. Summation will be assumed when the lower indices are repeated on the same side of the equations. The symbol : here stands for the double contracted product of tensors. Under such a stress state, the single crystal will induce a strain rate tensor (normalized by the reference shear rate):

$$D_{ij}=(1/2)\Sigma_s(m^s{}_{ij}+m^s{}_{ji})(m^s{}_{kl}S_{kl}/\tau_0{}^s)^{m'} \qquad (6)$$

When the five independent components of $\underset{\sim}{D}$ are applied, the set of non-linear equations (6) can be inverted by a Newton-Raphson method, since the existence of a convex viscoplastic potential ensures unicity of the solution. Divergence is prevented by rescaling the stress correction whenever it is necessary, i.e. when the stress tensor generates too large resolved stresses. This method has proved to be very efficient and safe.

First, theTaylor (Full Constraints, FC) assumption is made, namely:

$$\underset{\sim}{D}=\underset{\sim}{\bar{D}} \qquad (7)$$

i.e. uniformity of the strain rate throughout the polycrystal. Knowing the macroscopic strain rate, it is possible to invert equations (6) for each grain and average out the stress over the sample volume:

$$\underset{\sim}{\bar{S}}=\langle f(g)\underset{\sim}{S}(g)\rangle \qquad (8)$$

where g represents a grain orientation, f(g) the volume fraction of grains having same orientation as grain g, and the symbol <> represents a volume average. When the Relaxed Constraints (RC) assumption is made, equ.(6) are inverted only in the subspace of the imposed strain components, keeping the complementary imposed stresses constant. The orientation changes responsible for the induced texture are calculated with a method due to Honneff and Mecking [5], and used by Canova [6] and

Canova et al.[7], which relies on the fact that the sheet plane and major principal straining direction are considered as fixed in space, throughout the material.

Other informations are needed for the instability problem, namely the macroscopic hardening and the macroscopic viscosity tensors. They can be obtained as follows: a (space or time) variation in strain rate can be associated with an orientation change, a stress change and a change in the set of reference shear stresses. This can be expressed as:

$$\begin{aligned} dD_{ij} = & \Sigma_s dR^s_{ij}(\tau^s/\tau_o{}^s)^{m'} \\ & + m'\Sigma_s R^s_{ij}(\tau^s/\tau_o{}^s)^{m'-1} dR^s_{kl} S_{kl}/\tau_o{}^s \\ & + m'\Sigma_s R^s_{ij} R^s_{kl}(\tau^s/\tau_o{}^s)^{m'-1} dS_{kl}/\tau_o{}^s \\ & - m'\Sigma_s R^s_{ij}(\tau^s/\tau_o{}^s)^{m'} d\tau_o{}^s/\tau_o{}^s \end{aligned} \tag{9}$$

where the tensor $\underset{\sim}{R}^s$ is the symmetrical part of the tensor $\underset{\sim}{m}^s$. This equation is to be understood in a fixed external frame. The changes in the $\underset{\sim}{R}^s$ tensors are associated to orientation changes, i.e. to strain increments, and so are changes in the set of reference stresses. It will be demonstrated in Appendix A that they can be written as:

$$dR^s_{ij} = U^s_{ijkl} d\varepsilon_{kl}$$

and (10)

$$d\tau^s_o = V^s_{kl} d\varepsilon_{kl}$$

so that equ.(9) can be expressed as:

$$\begin{aligned} dD_{ij} = & \Sigma_s U^s_{ijkl}(\tau^s/\tau^s_o)^{m'} d\varepsilon_{kl} \\ & + m'\Sigma_s R^s_{ij}(\tau^s/\tau_o{}^s)^{m'-1} U^s_{mnkl} S_{mn} d\varepsilon_{kl}/\tau^s_o \\ & + m'\Sigma_s R^s_{ij} R^s_{kl}(\tau^s/\tau_o{}^s)^{m'-1} dS_{kl}/\tau^s_o \\ & - m'\Sigma_s R^s_{ij}(\tau^s/\tau_o{}^s)^{m'} V^s_{kl} d\varepsilon_{kl}/\tau_o{}^s \end{aligned} \tag{11}$$

which can be separated into terms depending on $d\underset{\sim}{\varepsilon}$ and a term depending on $d\underset{\sim}{S}$:

$$dD_{ij} = \eta^{-1}_{ijkl} dS_{kl} - H^*_{ijkl} d\varepsilon_{kl} \tag{12}$$

where $\underset{\sim}{\eta}$ represents a viscosity, i.e. the instantaneous stress change due to a strain rate change, whereas $\underset{\sim}{H}^*$ is a hardening tensor in which both texture and microscopic hardening are included. The tensor $\underset{\sim}{\eta}$ can, in general, be inverted [8], so that equ.(12) can be rewritten in the form:

$$dS_{ij} = \eta_{ijkl} dD_{kl} + H_{ijkl} d\varepsilon_{kl} \tag{13}$$

The elasticity is here entirely neglected, which seems legitimate since the material reaches large strains and is fully plastically loaded in the bulk and in the groove.

Here again, when the FC assumption is used, uniformity of $d\underset{\sim}{\varepsilon}$ and $d\underset{\sim}{D}$ are prescribed so that the macroscopic $\underset{\sim}{\eta}$ and $\underset{\sim}{H}$ tensors are obtained by volume averaging of the local corresponding tensors. When RC conditions are taken, the large flat planes of the grains usually coincide with the sheet plane, so that the in-plane strain components are still prescribed to be uniform, and volume averaging of the $\underset{\sim}{\eta}$ and $\underset{\sim}{H}$ tensors can be safely done in the 3D subspace in which the problem is studied.

The analysis does not presume any existence of orthotropy axes. Only the assumption that the sheet plane is a mirror plane for the texture and therefore for the anisotropy is used. This implies that the stress subspace $(\pi, S_{12})$ is closed [9], i.e. any strain rate having components in that subspace will induce a stress having zero components in the complementary subspace; it means, in particular, that there is no need to look for non-zero components $D_{t3}$ and $D_{n3}$ to fulfill the equs.(1), since they will turn out to be zero by symmetry.

It will also be assumed that the microscopic law of work-hardening is unique and independent of the strain path, which is a fairly good approximation according to Tomé et al.[10]. In this work, a simple Voce law has been taken to describe the isotropic hardening of the slip systems, which is expressed in differential form:

$$d\tau_0{}^s=\theta_0(\tau_1+\tau_{00}-\tau_0)\Sigma|d\gamma^s|/\tau_1 \quad (14)$$

where $\theta_0$ is the initial hardening rate, $\tau_0$, the current reference stress, and $\tau_1$ is so that its sum with the initial stress $\tau_{00}$ is the saturation stress.

The next section will concern the application of the Defect analysis.

## THE DEFECT ANALYSIS

The groove generated artificially is a material line whose angle varies according to the geometrical relation:

$$\tan(\Psi)=\tan(\Psi_0).\exp((1-\rho)E_1) \quad (15)$$

where $\Psi$ and $\Psi_0$ are respectively the current and initial groove angle (fig.1), $E_1$ the accumulated major strain, and $\rho$ the bulk strain rate ratio as defined in equ.(3). At each strain step, the strain rate is adjusted to fulfill the equs.(1), which are then taken as a set of non-linear equations in $D_{nn}$, $D_{tt}$ and $D_{nt}$. The critical strain is considered to be attained when the ratio of equivalent strain rates in the groove and the bulk equals the arbitrary value of 3. The correct groove angle is the one minimizing the critical strain. The initial groove angles have been taken from 0° to 90° with increments of 5°. The initial defect size is taken as 1% and the rate sensitivity equals 0.02. When the texture is kept isotropic along a given strain path, and for a microscopic hardening defined with:

$$\tau_{00}=1.;\ \tau_1=2.;\ \theta_0=10. \quad (16)$$

the F.L.D. obtained using those data is shown on fig.2. We will restrict the application of Defect Theory to the study of grain shape and induced texture effects.

As explained above, it is possible to get the limit strains in the case of flat grains, by imposing to each grain the stress components:

$$S_{13}=S_{23}=0. \quad (17)$$

and deriving the missing stress components in the subspace where the strain rate components are known. No noticeable difference has been found for the case of uniaxial and plane strain tension. A significant influence of grain shape appears in the expansion region, as shown on fig.3. The evolution of the ratio of equivalent strain rates clearly indicates a loss of ductility due to grain shape effects. This effect has already been pointed out by Kocks et al.[4]. It is due to the fact that flat grains induce a vertex at the equibiaxial stretching loading point (fig.4a). The appearance of that vertex is not an artifact of the RC theory, since even a low interaction self-consistent scheme [11] does predict such a sharp curvature as shown on fig.4b.

When the texture is left free to develop during the strain path, again no noticeable difference is observed for $\rho=0$ . In equibiaxial expansion, the effect of the induced texture is not severe for the work-hardening described in equs.(16), as can be seen in fig.5a, but the effect is more pronounced for higher work-hardening materials (fig.5b). Although there is here no grain shape effect, the texture induces a change in the polycrystal yield surface, and particularly a sharper curvature at the loading point . The increasing ductility observed in uniaxial tension when the texture is free to develop is due to the fact that the FC tension texture exhibits a fairly large texture hardening due to the strengthening of the (111) texture component.

The next section will be devoted to an alternative way of deriving F.L.D.'s namely the Perturbation Theory.

## THE PERTURBATION METHOD

The main purpose of the Perturbation Method is to assume a certain additional inhomogeneity in the strain, strain rate and stress fields, see under which condition the material will allow such a fluctuation, and study the development of this perturbation, particularly whether its size will increase or not. This approach is commonly used for studying the stability of the homogeneous solution of a differential system of equations, however, it has not yet been applied to date to the sheet forming problem, except in the study of Dudzinski and Molinari [13]. The three equs.(1) can also be written as:

$$\begin{aligned} &\partial(h\sigma_{nn})/\partial X_n=0 \\ &\partial(h\sigma_{nt})/\partial X_n=0 \qquad\qquad (17) \\ &\partial(D_{tt})/\partial X_n=0 \end{aligned}$$

where the reference system ($X_n$, $X_t$, $X_3$) is defined in fig.1. For the present problem, we chose to 'vectorize' the tensors using the Lequeu notation [12]. A rank two symmetrical tensor ,e.g. $\underset{\sim}{Z}$, has a unique associated 5D vector, $\underset{\sim}{Z}$, so that:

$$\begin{aligned} &Z_1=Z_{33}\sqrt{3/2} \\ &Z_2=(Z_{22}-Z_{11})/\sqrt{2} \\ &Z_3=Z_{23}\sqrt{2} \qquad\qquad (18) \\ &Z_4=Z_{13}\sqrt{2} \\ &Z_5=Z_{12}\sqrt{2} \end{aligned}$$

This definition enables us to write:

$$\sigma_{nn}=-(S'_1\sqrt{3}+S'_2)/\sqrt{2}$$

$$\sigma_{nt}=S'_5/\sqrt{2}$$

$$D_{tt}=(D'_2-D'_1/\sqrt{3})/\sqrt{2} \tag{19}$$
$$dh=hd\varepsilon_1\sqrt{2/3}$$

where the $S'_i$ are the vector components of the deviatoric stress tensor in groove axes, $D'_i$ and $d\varepsilon_i$ the vector components of the strain rate and strain increment tensors in groove axes as well. We assume that the strain field has a variation of the kind:

$$\underset{\sim}{\varepsilon}=\underset{\sim}{\varepsilon}^{0}+\partial\underset{\sim}{\varepsilon}^{*}.\exp(\lambda t).\exp[i(\xi_1X_1+\xi_2X_2)] \tag{20}$$
$$=\underset{\sim}{\varepsilon}^{0}+\partial\underset{\sim}{\varepsilon}(X,t)$$

where $\lambda$ is not known a-priori, and $\xi_i$ characterizes the periodicity of the inhomogeneity with respect to the $X_i$ axes, it is not known a-priori either. $\underset{\sim}{\varepsilon}^{0}$ is the homogeneous value of the strain, and $\partial\underset{\sim}{\varepsilon}^{*}$ an arbitrary initial amplitude. The exact definition of the strain is here not important since only strain increments (with time or space) will be used. When $\lambda$ is found to be negative, the perturbation decays in magnitude leading to homogeneous stable flow, while a positive value, on the contrary, leads to the onset of instability. The strain rate field, using equ.(20) will be of the form:

$$\underset{\sim}{D}=\underset{\sim}{D}^{0}+\lambda\partial\underset{\sim}{\varepsilon}(X,t) \tag{21}$$

and therefore, the stress increment in the fixed principal strain axes will be:

$$dS_i=dS^{0}_i+(\lambda\eta_{ij}+H_{ij}).\partial\varepsilon_j(X,t) \tag{22}$$

here the fourth rank tensors $\underset{\sim}{\eta}$ and $\underset{\sim}{H}$ have been changed into two index matrices by the Lequeu transformation. Here $d\underset{\sim}{S}^{0}$ is the homogeneous stress increment field which turns out to be zero when space variations are taken.

It is possible, then, when substituting the relations (20), (21) and (22) into the non-linear equations (17), and selecting the first order terms in $\partial\underset{\sim}{\varepsilon}$, to rewrite the differential system, equs.(17) into the form:

$$(A_{ij}+\lambda B_{ij})\partial\varepsilon_j=0 \tag{23}$$

where:

$$\begin{aligned}
&A_{1j}=-(cS_2+sS_5+S_1\sqrt{3})\sqrt{2/3}\partial_{1j}-cH_{2j}-sH_{5j}-H_{1j}\sqrt{3}\\
&A_{2j}=(-sS_2+cS_5)\sqrt{2/3}\partial_{1j}-sH_{2j}+cH_{5j}\\
&A_{3j}=-\partial_{1j}(1/\sqrt{3})+c\partial_{2j}+s\partial_{3j}\\
&B_{1j}=-c\eta_{2j}-s\eta_{5j}-\eta_{1j}\sqrt{3}\\
&B_{2j}=-s\eta_{2j}+c\eta_{5j}\\
&B_{3j}=0
\end{aligned} \tag{24}$$

with:

$$s=\sin(2\Psi);\ c=\cos(2\Psi)$$

It can be seen that, for a non-zero perturbation to exist, it is necessary that the matrix $\underset{\sim}{A}+\lambda\underset{\sim}{B}$ be singular which provides a second order algebraic equation in $\lambda$. The matrix considered depends not only on $\underset{\sim}{S}$, $\underset{\sim}{\eta}$ and $\underset{\sim}{H}$ but also on the angle $\Psi$. The groove angle of interest is the one providing the maximum positive root $\lambda^{+}$, since it will be the most destabilizing one. The corresponding other root is called $\lambda^{-}$. It is not rigorous to consider the flow unstable as soon as $\lambda^+$ is positive since the homogeneous solution in strain is not constant. Instability will in fact occur when $\lambda^+$ is significantly positive, which leaves a degree of arbitrariness in the choice of the critical strain.

This analysis has been firstly applied to ideal rolling texture components. On figs.7a to e are shown the variations of $\lambda^+$ and $\lambda^-$ for each texture component and the three loading paths ($\rho$=-1/2, 0 and 1). On fig.6 are the corresponding F.L.D.'s compared with the ones obtained by the Defect analysis. The level of $\lambda^+$ corresponding to instability has been chosen so that the critical strain in uniaxial tension are identical to those found by the Defect theory. It can be seen, on fig.6 that a good general agreement is found between the two theories, the largest deviation being about 3% in equibiaxial expansion for the Cu component. A low ductility is here found in the expansion region, since only the four orthotropic ideal orientations of each texture component have been taken, providing therefore vertex effects.

When this analysis is applied to the polycrystal case, and the critical strain defined as previously, the calculated F.L.D shows a perfect agreement in the $\rho<0$ region between the two theories (fig.7), but a very large difference appears in the expansion region. The relatively good agreement obtained with the ideal texture components may mean that the curvature of the yield surface obtained through $\eta$ is not well described. This tensor is positive and symmetrical, and it represents therefore an ellipsoid which is tangent to the yield surface at the loading point. However, when the comparison is made between the real local curvature and the one simulated by using the tensor $\eta$, a good correspondence is found.

The Perturbation method, therefore, can not yet be applied safely in expansion, but seems to give good results otherwise. It is important to note that the computing time used by this method is less than 1% of the time spent by the Defect theory, and therefore may be very useful in the future.


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## CONCLUSIONS

The forming ability of a viscoplastic polycrystalline material has been studied using Taylor (FC) and relaxed-constraints (RC) assumptions to describe the average behavior. The Defect theory has been used to investigate particularly the effects of grain shape and induced texture on the ductility, and a new way of getting F.L.D.'s has been proposed by using the Perturbation theory.

We found a dramatic grain shape influence on the ductility in expansion, but no noticeable influence for the cases $\rho=0$ to -1/2. This has been attributed to the appearance of a vertex at the loading point which enables the groove material to deform into a plane strain mode, i.e. to localize.

The effect of the induced texture is shown to be more important the larger the work-hardening rate, particularly in equibiaxial expansion.

The forming problem has also been investigated by using the Perturbation theory, which consists into imposing an inhomogeneous strain field and studying whether the perturbation will persist leading to localization or not. It has the major advantage of taking less than 1% of the computing time used by the former theory. The F.L.D.'s obtained for the ideal rolling texture components do coincide well with the ones obtained by the Defect theory, some small discrepancies appearing in the expansion case. Very good agreement is also observed in the $\rho<0$ region for the polycrystal case, whereas a very large discrepancy appears in the equibiaxial tension case. This misfit between theories in that case is not attributed to inaccuracies in the description of the yield surface curvature, and does not yet have a clear explanation.

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## APPENDIX A

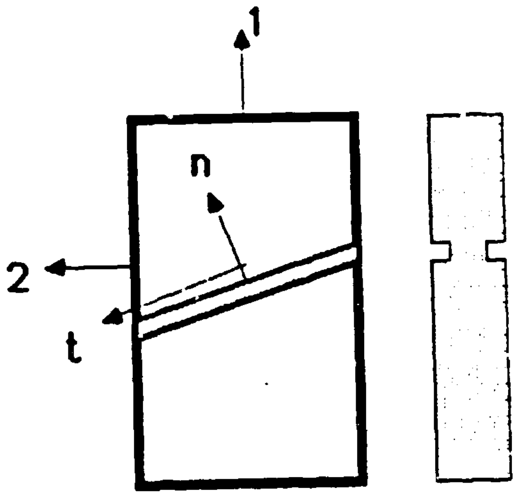


Figure 1: Geometry of the sample used in Defect Theory to study the material ductility.

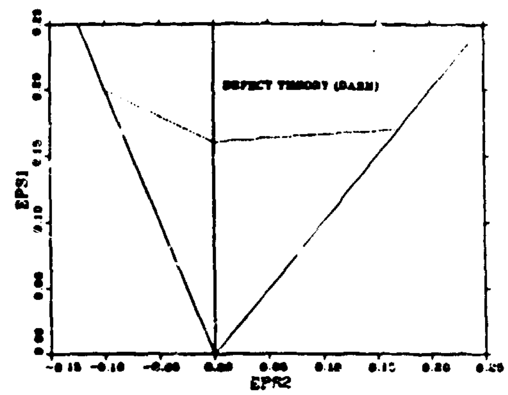


Figure 2: F.L.D. obtained for an isotropic polycrystal, having microscopic hardening parameters defined in equ.(16).

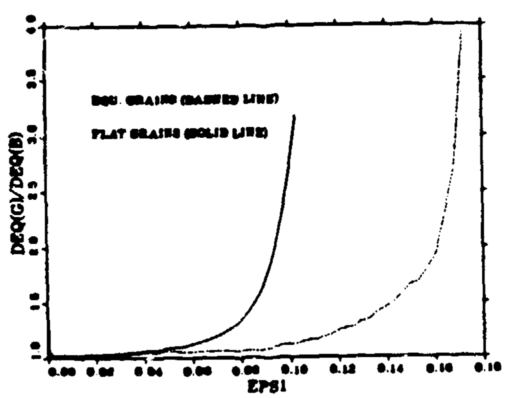


Figure 3: Equivalent strain rate ratios obtained for an isotropic polycrystal in equibiaxial expansion. Note the decrease in ductility obtained for flat grains.

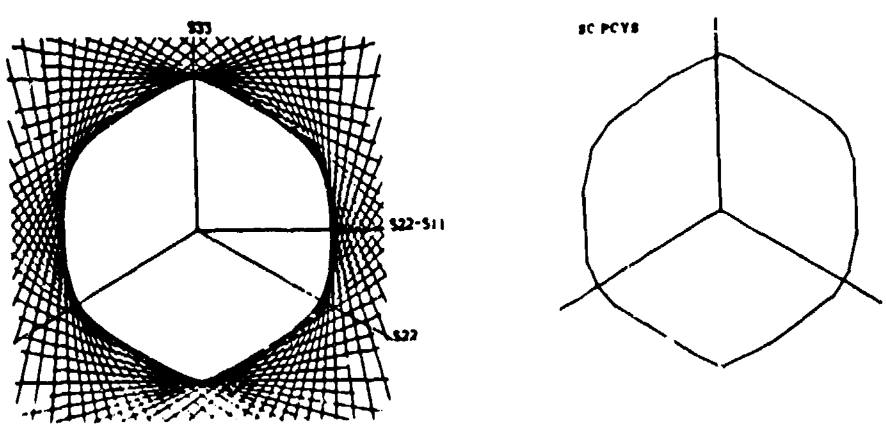


Figure 4: π plane yield surfaces obtained for an isotropic polycrystal having flat grains. RC theory (a) and a self-consistent viscoplastic (b) theory have been applied.

Equ.(6) can be written in the form:

$$D=\Sigma(\tau^s/\tau_o{}^s)^{m'-1}(R^s x R^s):S/\tau_o{}^s \qquad \text{(A-1)}$$

or $\quad D_{ij}=F_{ijkl}S_{kl}$

where the secant tensor $\underset{\sim}{F}$ can be calculated when the non-linear equs(6) have been inverted and $\underset{\sim}{S}$ obtained. It is then possible to get the microscopic shearing rates:

$$\dot{\gamma}^s/\dot{\gamma}_o=(\tau^s/\tau^s{}_o)^{m'-1}\underset{\sim}{R}:\underset{\sim}{F}^{-1}:\underset{\sim}{D}/\tau_o{}^s \qquad \text{(A-2)}$$

or: $\quad =\underset{\sim}{P}^s:\underset{\sim}{D}$

In FC conditions, the total spin is zero so that the lattice spin is the opposite to the plastic spin calculated as follows:

$$\omega^*{}_{ij}=(1/2)\Sigma(m^s{}_{ij}-m^s{}_{ji})P^s{}_{kl}D_{kl} \qquad \text{(A-3)}$$

$$=T_{ijkl}D_{kl}$$

The change in the $\underset{\sim}{R}^s$ tensors can therefore be written as:

$$dR^s{}_{ij}=d\omega^*{}_{ik}R^s{}_{kj}-R^s{}_{ik}d\omega^*{}_{kj} \qquad \text{(A-4)}$$

which can be expressed as:

$$dR^s{}_{ij}=(T_{ikmn}R^s{}_{kj}-R^s{}_{ik}T_{kjmn})d\varepsilon_{mn} \qquad \text{(A-5)}$$

$$=U^s{}_{ijkl}d\varepsilon_{kl}$$

The microscopic hardening, when restricted to isotropic hardening of the slip systems is written:

$$d\tau^s{}_o=h\Sigma|d\gamma^s|=h\Sigma\alpha_s d\gamma^s \qquad \text{(A-6)}$$

$$=h\Sigma\alpha_s\underset{\sim}{P}^s:d\underset{\sim}{\varepsilon}=\underset{\sim}{V}^s:d\underset{\sim}{\varepsilon}$$

where h is the hardening rate defined in equ.(14), and $\alpha^s$ the sign of $d\gamma^s$.

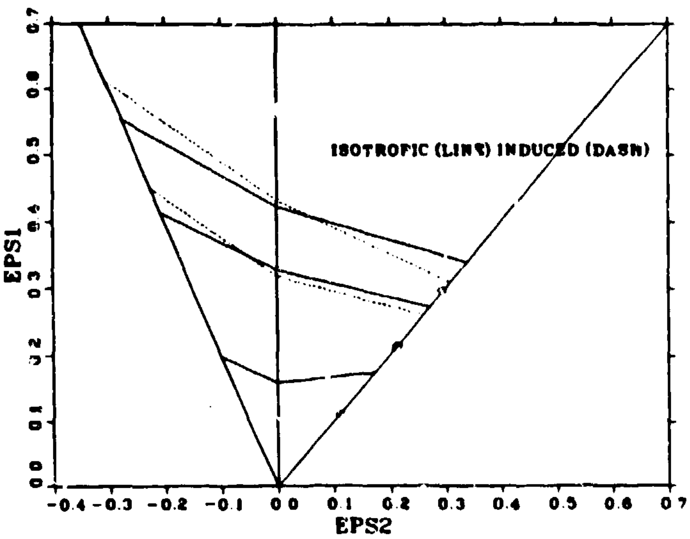


Figure 5: F.L.D.'s predicted using an initially isotropic polycrystal, and by leaving the texture evolve (dashed curves) or not (full lines) during the forming process, for three levels of microscopic hardening.

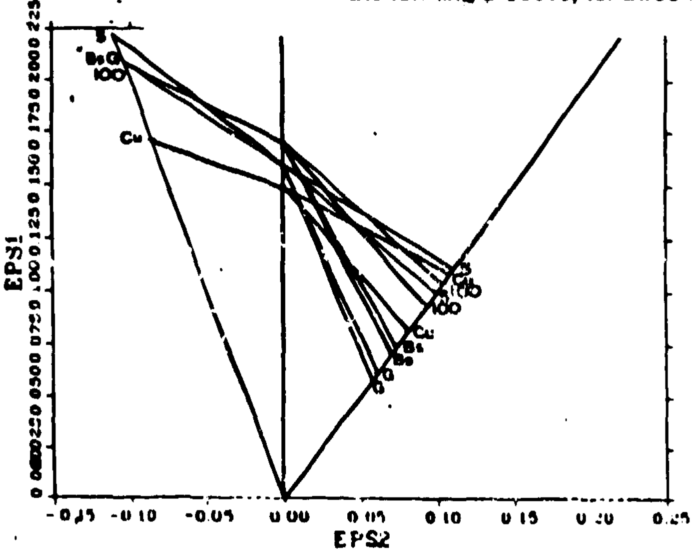


Figure 6: Critical strains obtained on some rolling texture components by the two theories. Note the good correspondance for $\rho<0$, and the relatively good qualitative agreement for $\rho>0$.

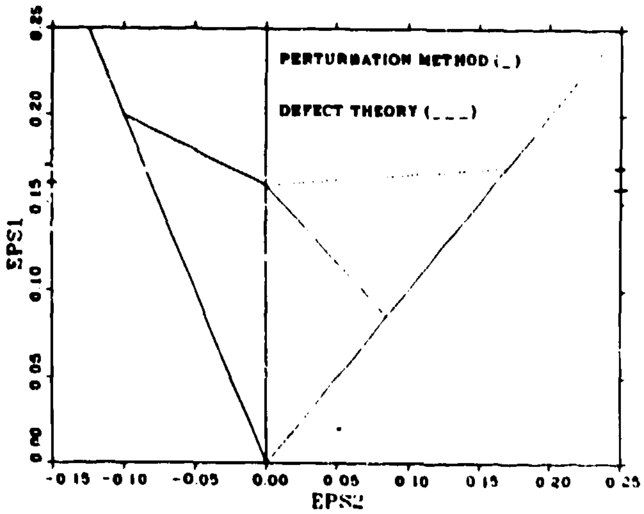


Figure 7: F.L.D's obtained on an isotropic polycrystal using both theories. The agreement is good for $\rho<0$ but rather poor in expansion.